

Observable Lepton Flavor Symmetry at LHC

Ernest Ma

*Department of Physics and Astronomy, University of California,
Riverside, California 92521, USA*

Abstract

I discuss a model of lepton flavor symmetry based on the non-Abelian finite group T_7 and the gauging of $B - L$, which has a residual Z_3 symmetry in the charged-lepton Yukawa sector, allowing it to be observable at the Large Hadron Collider (LHC) from the decay of the new Z' gauge boson of this model to a pair of scalar bosons which have the unusual highly distinguishable final states $\tau^-\tau^-\mu^+e^+$.

Talk at the “International Conference on Flavor Physics in the LHC Era,” Singapore (November 2010).

1 A Short History of A_4

In 1978, soon after the putative discovery of the third family of leptons and quarks, it was conjectured by Cabibbo[1] and Wolfenstein[2] independently that the 3×3 lepton mixing matrix may be given by

$$U_{\nu}^{CW} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}, \quad (1)$$

where $\omega = \exp(2\pi i/3) = -1/2 + i\sqrt{3}/2$. This implies $\theta_{12} = \theta_{23} = \pi/4$, $\tan^2 \theta_{13} = 1/2$, and $\delta_{CP} = \pm\pi/2$. Thirty years later, we know that they were not completely correct, but their bold conjecture illustrated the important point that not everyone expected small mixing angles in the lepton sector as in the quark sector. The fact that neutrino mixing turns out to involve large angles should not have been such a big surprise.

In 2001, Ma and Rajsekaran[3] showed that the non-Abelian discrete symmetry A_4 allows $m_{e,\mu,\tau}$ to be arbitrary, and yet $\sin^2 2\theta_{atm} = 1$, $\theta_{e3} = 0$ can be obtained. In 2002, Babu, Ma, and Valle[4] showed how $\theta_{13} \neq 0$ can be radiatively generated in A_4 with the prediction that $\delta_{CP} = \pm\pi/2$, i.e. maximum CP violation.

In 2002, Harrison, Perkins, and Scott[5], after abandoning their bimaximal and trimaximal hypotheses, proposed the tribimaximal mixing matrix, i.e.

$$U_{\nu}^{HPS} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix} \sim (\eta_8, \eta_1, \pi^0), \quad (2)$$

which is easy to remember in terms of the meson nonet. This means that $\sin^2 2\theta_{atm} = 1$, $\tan^2 \theta_{sol} = 1/2$, $\theta_{e3} = 0$.

In 2004, I showed[6] that tribimaximal mixing may be obtained in A_4 , with

$$U_{CW}^\dagger M_\nu U_{CW} = \begin{pmatrix} a+2b & 0 & 0 \\ 0 & a-b & d \\ 0 & d & a-b \end{pmatrix}, \quad (3)$$

in the basis that M_l is diagonal. At that time, the Sudbury Neutrino Observatory (SNO) data gave $\tan^2 \theta_{sol} = 0.40 \pm 0.05$, but it was changed in early 2005 to 0.45 ± 0.05 . Thus tribimaximal mixing and A_4 became part of the lexicon of the neutrino theorist.

After the 2005 SNO revision, two A_4 models quickly appeared. (I) Altarelli and Feruglio[7] proposed

$$U_{CW}^\dagger M_\nu U_{CW} = \begin{pmatrix} a & 0 & 0 \\ 0 & a & d \\ 0 & d & a \end{pmatrix}, \quad (4)$$

i.e. $b = 0$, and (II) Babu and He[8] proposed

$$U_{CW}^\dagger M_\nu U_{CW} = \begin{pmatrix} a' - d^2/a' & 0 & 0 \\ 0 & a' & d \\ 0 & d & a' \end{pmatrix}, \quad (5)$$

i.e. $d^2 = 3b(b - a)$.

The *challenge* is to prove experimentally that A_4 or some other discrete symmetry is behind neutrino tribimaximal mixing. If A_4 is realized by a renormalizable theory at the electroweak scale, then the extra Higgs doublets required will bear this information. Specifically, A_4 breaks to the residual symmetry Z_3 in the charged-lepton sector, and all Higgs Yukawa interactions are determined in terms of lepton masses. This notion of *lepton flavor triality*[9] may be the key to such a proof, but these exotic Higgs doublets are very hard to see at the LHC.

2 Frobenius Group T_7

The tetrahedral group A_4 (12 elements) is the smallest group with a real $\underline{3}$ representation. The Frobenius group T_7 (21 elements) is the smallest group with a pair of complex $\underline{3}$ and $\underline{3}^*$ representations. It is generated by

$$a = \begin{pmatrix} \rho & 0 & 0 \\ 0 & \rho^2 & 0 \\ 0 & 0 & \rho^4 \end{pmatrix}, \quad b = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \quad (6)$$

where $\rho = \exp(2\pi i/7)$, so that $a^7 = 1$, $b^3 = 1$, and $ab = ba^4$. It has been considered by Luhn, Nasri, and Ramond[10], Hagedorn, Schmidt, and Smirnov[11], as well as King and Luhn[12]. The character table of T_7 (with $\xi = -1/2 + i\sqrt{7}/2$) is given by

class	n	h	χ_1	$\chi_{1'}$	$\chi_{1''}$	χ_3	χ_{3^*}
C_1	1	1	1	1	1	3	3
C_2	7	3	1	ω	ω^2	0	0
C_3	7	3	1	ω^2	ω	0	0
C_4	3	7	1	1	1	ξ	ξ^*
C_5	3	7	1	1	1	ξ^*	ξ

Table 1: Character table of T_7 .

The group multiplication rules of T_7 include

$$\underline{3} \times \underline{3} = \underline{3}^*(23, 31, 12) + \underline{3}^*(32, 13, 21) + \underline{3}(33, 11, 22), \quad (7)$$

$$\begin{aligned} \underline{3} \times \underline{3}^* &= \underline{3}(21^*, 32^*, 13^*) + \underline{3}^*(12^*, 23^*, 31^*) + \underline{1}(11^* + 22^* + 33^*) \\ &+ \underline{1}'(11^* + \omega 22^* + \omega^2 33^*) + \underline{1}''(11^* + \omega^2 22^* + \omega 33^*). \end{aligned} \quad (8)$$

Note that $\underline{3} \times \underline{3} \times \underline{3}$ has two invariants and $\underline{3} \times \underline{3} \times \underline{3}^*$ has one invariant. These serve to distinguish T_7 from A_4 and $\Delta(27)$.

3 $U(1)_{B-L}$ Gauge Extension with T_7

Recently, the following model has been proposed by Cao, Khalil, Ma, and Okada[13]: Under T_7 , let $L_i = (\nu, l)_i \sim \underline{3}$, $l_i^c \sim \underline{1}, \underline{1}', \underline{1}''$, $\Phi_i = (\phi^+, \phi^0)_i \sim \underline{3}$, which means that $\tilde{\Phi} = (\bar{\phi}^0, -\phi^-)_i \sim \underline{3}^*$. The Yukawa couplings $L_i l_j^c \tilde{\Phi}_k$ generate the charged-lepton mass matrix

$$M_l = \begin{pmatrix} f_1 v_1 & f_2 v_1 & f_3 v_1 \\ f_1 v_2 & \omega^2 f_2 v_2 & \omega f_3 v_2 \\ f_1 v_3 & \omega f_2 v_3 & \omega^2 f_3 v_3 \end{pmatrix} = U_{CW}^\dagger \begin{pmatrix} f_1 & 0 & 0 \\ 0 & f_2 & 0 \\ 0 & 0 & f_3 \end{pmatrix} \sqrt{3} v, \quad (9)$$

if $v_1 = v_2 = v_3 = v$ as in the original A_4 proposal.

Let $\nu_i^c \sim \underline{3}^*$, then the Yukawa couplings $L_i \nu_j^c \Phi_k$ are allowed, with

$$M_D = f_D v \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}. \quad (10)$$

Note that Φ and $\tilde{\Phi}$ have $B - L = 0$.

Now add the neutral Higgs singlets $\chi_i \sim \underline{3}$ and $\eta \sim \underline{3}^*$, both with $B - L = -2$. Then there are two Yukawa invariants: $\nu_i^c \nu_j^c \chi_k$ and $\nu_i^c \nu_j^c \eta_k$. Note that $\chi_i^* \sim \underline{3}^*$ is not the same as $\eta_i \sim \underline{3}^*$ because they have different $B - L$. This means that both $B - L$ and the complexity of T_7 are required for this scenario. The heavy Majorana mass matrix for ν^c is then

$$M = h \begin{pmatrix} u_2 & 0 & 0 \\ 0 & u_3 & 0 \\ 0 & 0 & u_1 \end{pmatrix} + h' \begin{pmatrix} 0 & u'_3 & u'_2 \\ u'_3 & 0 & u'_1 \\ u'_2 & u'_1 & 0 \end{pmatrix} = \begin{pmatrix} A & 0 & B \\ 0 & A & 0 \\ B & 0 & A \end{pmatrix}, \quad (11)$$

where $A = hu_1 = hu_2 = hu_3$ and $B = h'u'_2$ with $u'_1 = u'_3 = 0$ have been assumed, i.e. χ_i breaks in the (1,1,1) direction, whereas η_i breaks in the (0,1,0) direction. This is the $Z_3 - Z_2$ misalignment also used in A_4 models. The seesaw neutrino mass matrix is now

$$M_\nu = -M_D M^{-1} M_D^T = \frac{-f_D^2 v^2}{A^3 - AB^2} \begin{pmatrix} A^2 - B^2 & 0 & 0 \\ 0 & A^2 & -AB \\ 0 & -AB & A^2 \end{pmatrix}, \quad (12)$$

i.e. the two-parameter tribimaximal form proposed by Babu and He, but without the auxiliary $Z_4 \times Z_3$ symmetry assumed there. Two limiting cases are (I) normal hierarchy ($d = -a$): $m_1 = m_2 = 0$, $m_3 = 2a$, and (II) inverted hierarchy ($d = 2a$): $m_1 = 3a$, $m_2 = -3a$, $m_3 = -a$, with the effective mass in neutrinoless double beta decay given by $m_{ee} = a = \sqrt{\Delta m_{atm}^2 / 8} = 0.02 \text{ eV}$.

4 Higgs Structure

In the charged-lepton Yukawa sector, i.e. $L_i l_j^c \tilde{\Phi}_k$, a residual Z_3 symmetry exists so that linear combinations of Φ_k become $\phi_0, \phi_1, \phi_2 \sim 1, \omega, \omega^2$ together with $e, \mu, \tau \sim 1, \omega^2, \omega$. Their

interactions are given by

$$\begin{aligned}
\mathcal{L}_Y &= (\sqrt{3}v)^{-1}[m_\tau \bar{L}_\tau \tau_R + m_\mu \bar{L}_\mu \mu_R + m_e \bar{L}_e e_R]\phi_0 \\
&+ (\sqrt{3}v)^{-1}[m_\tau \bar{L}_\mu \tau_R + m_\mu \bar{L}_e \mu_R + m_e \bar{L}_\tau e_R]\phi_1 \\
&+ (\sqrt{3}v)^{-1}[m_\tau \bar{L}_e \tau_R + m_\mu \bar{L}_\tau \mu_R + m_e \bar{L}_\mu e_R]\phi_2 + H.c.
\end{aligned} \tag{13}$$

$$\tag{14}$$

As a result, the rare decays $\tau^+ \rightarrow \mu^+ \mu^+ e^-$ and $\tau^+ \rightarrow e^+ e^- \mu^-$ are allowed, but no others. For example, $\mu \rightarrow e \gamma$ is forbidden. Here $\phi_1^0, \bar{\phi}_2^0 \sim \omega$, mixing to form mass eigenstates $\psi_{1,2}^0 = (\phi_1^0 \pm \bar{\phi}_2^0)/\sqrt{2}$. Using

$$\frac{B(\tau^+ \rightarrow \mu^+ \mu^+ e^-)}{B(\tau \rightarrow \mu \nu \nu)} = \frac{m_\tau^2 m_\mu^2 (m_1^2 + m_2^2)^2}{m_1^4 m_2^4} < \frac{2.3 \times 10^{-8}}{0.174}, \tag{15}$$

the bound $m_1 m_2 / \sqrt{m_1^2 + m_2^2} > 22$ GeV (174 GeV/ $\sqrt{3}v$) is obtained. Hence the production of $\psi_{1,2}^0 \bar{\psi}_{2,1}^0$ at the LHC with final states $\tau^- e^+ \tau^- \mu^+$ and $\tau^+ \mu^- \tau^+ e^-$ would be indicative of this Z_3 flavor symmetry.

5 LHC Observations

The $\phi_{1,2}$ scalar doublets have $B - L = 0$, so they do not couple directly to the Z'_{B-L} gauge boson, but they can mix, after $U(1)_{B-L}$ breaking, with the χ and η singlets ($B - L = -2$) which do. Thus this model can be tested at the LHC by discovering Z' from $q\bar{q} \rightarrow Z' \rightarrow \mu^- \mu^+$ and looking for $Z' \rightarrow \psi_{1,2}^0 \bar{\psi}_{2,1}^0$ with the subsequent decays $\psi \rightarrow \tau^- e^+$ and $\bar{\psi} \rightarrow \tau^- \mu^+$. We assume for simplicity that $m_1 = m_2$, and take the LHC energy as 14 TeV, which is expected to be reached in a year or two.

Let $\Gamma_0 = g_{B-L}^2 m_{Z'}/12\pi$, then the partial decay widths of Z' are

$$\Gamma_q = (6)(3)(1/3)^2 \Gamma_0, \tag{16}$$

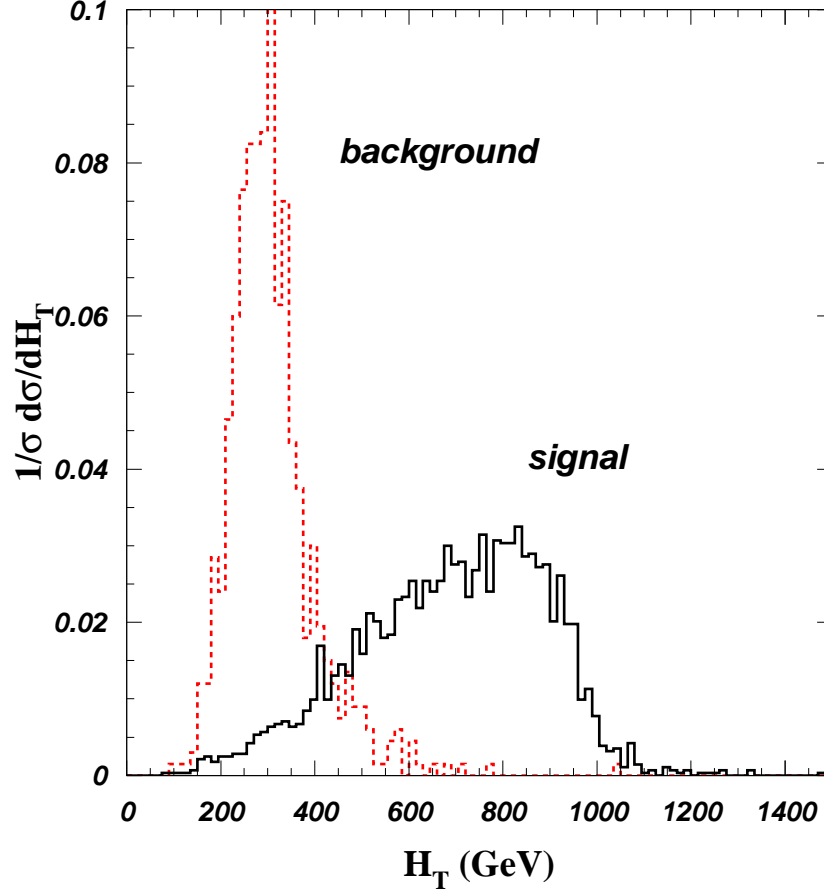


Figure 1: Normalized distribution of H_T .

$$\Gamma_l = (3)(-1)^2\Gamma_0, \quad (17)$$

$$\Gamma_\nu = (3)(-1)^2(1/2)\Gamma_0, \quad (18)$$

$$\Gamma_\psi \simeq (2)(-2)^2 \sin^2 \theta (1/4)\Gamma_0, \quad (19)$$

where $\sin \theta$ is an effective parameter accounting for the mixing of ψ to χ and η . The signature events are chosen to be $\tau^-\tau^-\mu^+e^+$ with τ^- decaying into $l^-(e^- \text{ or } \mu^-)$ plus missing energy. The background events yielding this signature come from

$$WWZ : pp \rightarrow W^+W^-Z, W^\pm \rightarrow l^\pm\nu, Z \rightarrow l^+l^-, \quad (20)$$

$$ZZ : pp \rightarrow ZZ, Z \rightarrow l^+l^-, Z \rightarrow \tau^+\tau^-, \tau^\pm \rightarrow l^\pm\nu\nu, \quad (21)$$

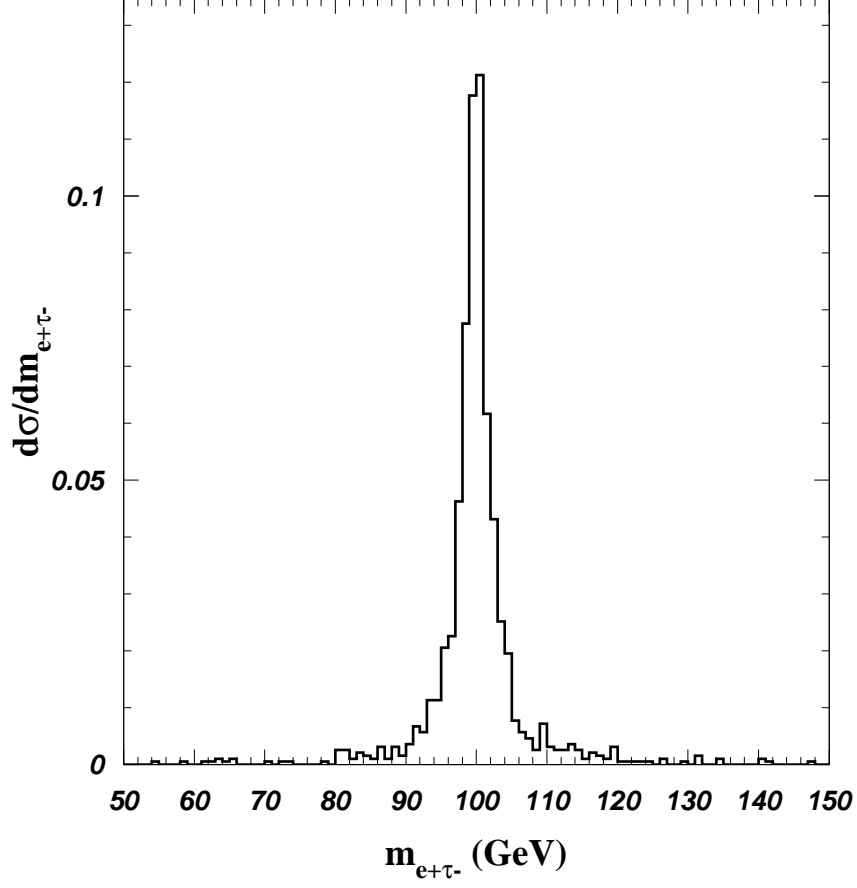


Figure 2: Distribution of the invariant mass of the e^+ and reconstructed τ^- pair.

$$t\bar{t} : pp \rightarrow t\bar{t} \rightarrow b(\rightarrow l^-)\bar{b}(\rightarrow l^+)W^+W^-, w^\pm \rightarrow l^\pm\nu, \quad (22)$$

$$Zb\bar{b} : pp \rightarrow Zb(\rightarrow l^-)\bar{b}(\rightarrow l^+), Z \rightarrow l^+l^-. \quad (23)$$

We require no jet tagging and consider only events with both e^+ and μ^+ in the final states. Our benchmark points for $m_{Z'}, m_\psi$ (in GeV) are (A), (1000,100), (B) (1500,100), (C) (1000,300), (D) (1500,300), with $g_{B-L} = g_2 = e/\sin\theta_W$, and $\sin^2\theta = 0.2$. We impose the following basic acceptance cuts:

$$p_{T,l}^{(1,2)} > 50 \text{ GeV}, \quad p_{T,l}^{(3,4)} > 20 \text{ GeV}, \quad |\eta_l| < 2.5, \quad (24)$$

$$\Delta R_{ij} \equiv \sqrt{(\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2} > 0.4, \quad \text{missing } E_T > 30 \text{ GeV}, \quad (25)$$

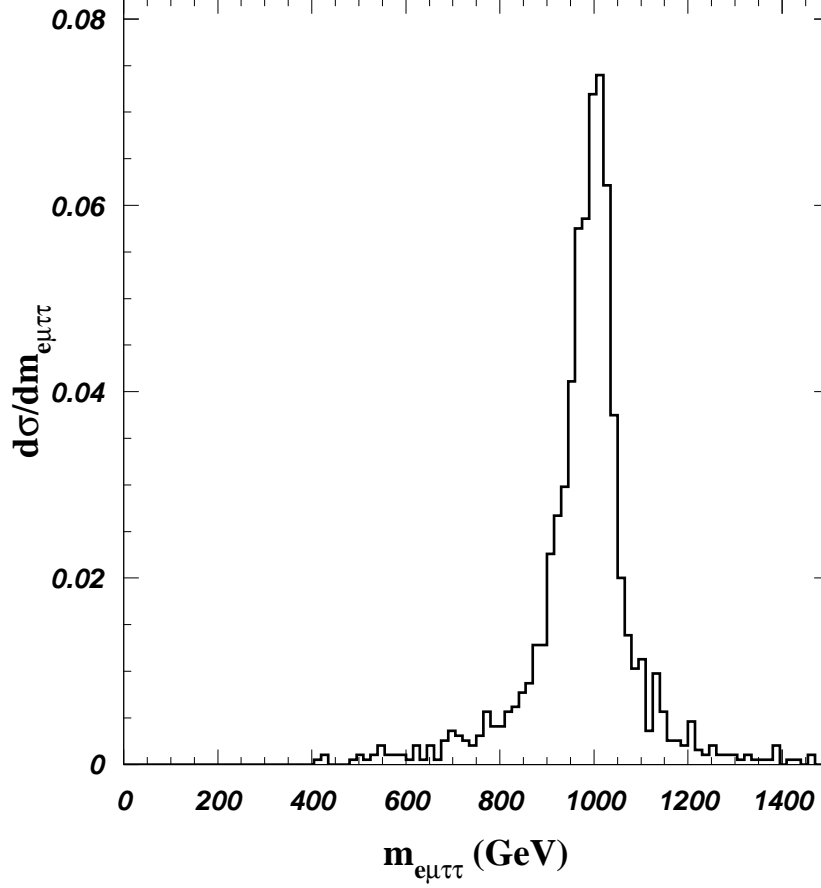


Figure 3: Distribution of the reconstructed Z' mass.

where ΔR_{ij} is the separation in the azimuthal angle (ϕ) – pseudorapidity (η) plane between i and j . We also model detector resolution effects by smearing the final-state energy. To further suppress the backgrounds, we require

$$H_T \equiv \sum_i p_{T,i} + \text{missing } E_T > 300 \text{ GeV}, \quad (26)$$

where i denotes the visible particles.

To reconstruct the scalar ψ , we adopt the collinear approximation that the l and ν 's from τ decays are parallel due to the τ 's large boost, coming from the heavy ψ . Denoting by x_{τ_i} the fraction of the parent τ energy which each observable decay particle carries, the

transverse momentum vectore are related by

$$\text{missing } E_T = (1/x_{\tau_1} - 1)\vec{p}_1 + (1/x_{\tau_2} - 1)\vec{p}_2. \quad (27)$$

When the decay products are not back-to-back, this gives two conditions for x_{τ_i} , with the τ momenta as \vec{p}_1/x_{τ_1} and \vec{p}_2/x_{τ_2} , respectively. We further require $x_{\tau_i} > 0$ to remove the unphysical solutions, and minimize $\Delta R_{e^+l^-}$ to choose the correct e^+l^- to reconstruct ψ and then Z' . In Table 2 we show the signal and background cross sections (in fb) for the benchmark cases (A) and (C).

	(A)	(C)	$t\bar{t}$	WWZ	ZZ	$Zb\bar{b}$
no cut	5.14	2.57	1.22	0.21	27.11	2.99
basic cut	1.46	1.05	0.16	0.02	0.0052	0.024
H_T cut	1.41	1.04	0.08	0.006	0.0	0.0
$x_\tau > 0$	0.69	0.52	0.015	0.002	0.0	0.0

Table 2: Signal and background cross sections (in fb) for $m_{Z'} = 1$ TeV and $m_\psi = 100$ GeV (A) and 300 GeV (C).

We show in Fig. 1 the H_T distribution in case (A) to demonstrate the separation of signal from background. We then show in Fig. 2 how the mass of ψ may be obtained from $e^+\tau^-$, and in Fig. 3 how the Z' mass may be reconstructed. The 5σ discovery contours for (A) to (D) are shown in Fig. 4 in the $m_{Z'} - \sin^2\theta$ plane.

6 Conclusion

Using the non-Abelian discrete symmetry T_7 together with the gauging of $B - L$, a simple renormalizable two-parameter model of the neutrino mass matrix is obtained with tribimaximal mixing. The charged-lepton Higgs Yukawa interactions are predicted completely and exhibit a residual Z_3 symmetry which is verifiable at the LHC. The signature is $pp \rightarrow Z' \rightarrow \psi\bar{\psi}$

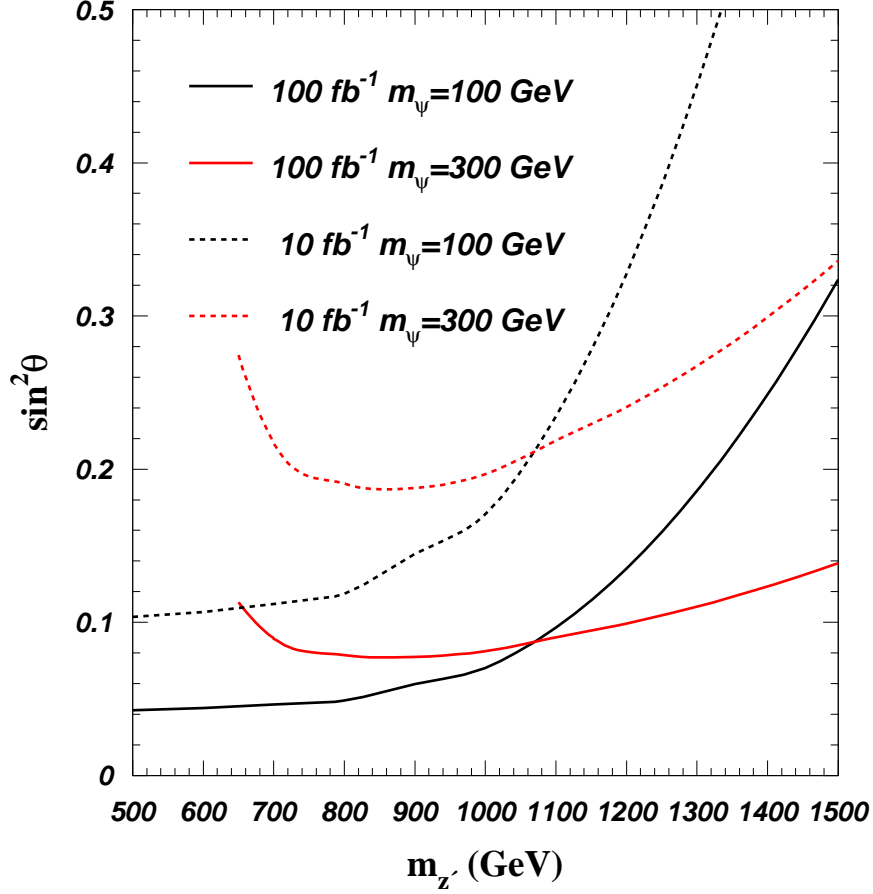


Figure 4: The 5σ discovery contours in the $m_{Z'} - \sin^2 \theta$ plane.

with the subsequent decays $\psi \rightarrow \tau^- e^+$ and $\bar{\psi} \rightarrow \tau^- \mu^+$. With 10 fb^{-1} at $E_{cm} = 14 \text{ TeV}$ and $m_{Z'} \sim 1 \text{ TeV}$, a 5σ discovery is expected.

Acknowledgements

I thank Harald Fritzsch, K. K. Phua, and the other organizers for their great hospitality and a stimulating conference. This work is supported in part by the U. S. Department of Energy under Grant No. DE-FG03-94ER40837.

References

- [1] N. Cabibbo, *Phys. Lett.* **B72**, 333 (1978).
- [2] L. Wolfenstein, *Phys. Rev.* **D18**, 958 (1978).
- [3] E. Ma and G. Rajasekaran, *Phys. Rev.* **D64**, 113012 (2001).
- [4] K. S. Babu, E. Ma, and J. W. F. Valle, *Phys. Lett.* **B552**, 207 (2003).
- [5] P. F. Harrison, D. H. Perkins, and W. G. Scott, *Phys. Lett.* **B530**, 167 (2002).
- [6] E. Ma, *Phys. Rev.* **D70**, 031901 (2004).
- [7] G. Altarelli and F. Feruglio, *Nucl. Phys.* **B720**, 64 (2005).
- [8] K. S. Babu and X.-G. He, arXiv:0507217 [hep-ph].
- [9] E. Ma, *Phys. Rev.* **D82**, 037301 (2010).
- [10] C. Luhn, S. Nasri, and P. Ramond, *Phys. Lett.* **B652**, 27 (2007).
- [11] C. Hagedorn, M. A. Schmidt, and A. Yu. Smirnov, *Phys. Rev.* **D79**, 036002 (2009).
- [12] S. F. King and C. Luhn, *JHEP* **0910**, 093 (2009).
- [13] Q.-H. Cao, S. Khalil, E. Ma, and H. Okada, arXiv:1009.5415 [hep-ph].